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Contemporary Theory of Firm Growth

by

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Contemporary Theory of Firm Growth[1]

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The main aim of this paper is to explain why large companies maximized their sales after World War II. I derive a model of firm growth from the degree of operating leverage formula. This model shows consistency between profit maximization and sales revenue maximization and implies that higher and higher sales revenue growth rates are needed in subsequent periods to reach the initial growth rate of profit. Because of this dependence in subsequent periods, it is increasingly difficult for firms to achieve the necessary growth rate of sales revenue. I also perform a sensitivity analysis on the decline in initial sales revenue, unit variable costs, and total fixed costs. I find that companies' situation in terms of profit maximization is deteriorating, as in subsequent periods ever-higher sales growth rates are necessary compared with the previous initial conditions. As a result, the company encounters demand or production capacity binding constraints, and the response of managers is to seek and apply new methods to increase the sales and market share domestically and internationally and not only to invest annually total depreciation but also to carry out large net investments to attain the necessary sales revenue growth rate. The model also supplies the microeconomic foundation for macroeconomics. In particular, it provides an explanation for the *momentum* and reversal phenomena.

I. Introduction

After World War II, it was observed that the objective of large firms was to maximize their sales revenues.[2] This required theoretical explanation. According to the neoclassical theory of an oligopolistic firm, maximum sales may be achieved for the production volume that generates losses. The phenomenon of maximizing sales by large firms is an anomaly in the context of the neoclassical paradigm. Baumol (1959) approached this matter by pointing out that the objective of a firm is not to increase sales in absolute terms but within a minimum profit constraint. This excluded an increase of sales up to the point at which it reaches the maximum. Marris (1963) was of the opinion that the objective of a firm is to maximize its sales growth rate within the managerial and financial constraints. Baumol (1962) reached the conclusion that the objective of a firm may be to maximize its sales growth rate. These papers pointed to a discrepancy between the interests of managers, whose aim is to maximize their usefulness, on the one hand and those of owners (shareholders), who expect profit maximization, on the other hand.

[1] The paper develops ideas presented in (Mielcarek 2004).

[2] For the sake of simplicity, the term sales will be used in the remaining part of the paper instead of sales revenue.

Uekusa and Caves (1976), Nakamura (1981), Kagono et al. (1985), Komiya (1987, 1992), Blinder (1992, 1993), and Tabeta and Wang (1996a, 1996b), among others, represent another wave of discussions on the sales maximization hypothesis. All these papers indicated that separating the management from the owners in joint stock companies creates conditions conducive to management autonomy. The objective may be to maximize sales or to maximize profit, depending on the conditions. According to Kagono et al. (1985), the main objective of Japanese firms is to maximize the sales growth rate and market share. American corporations, however, are more focused on short-term returns on investment and capital gains. Blinder (1993) created a model according to which firms that maximize sales have an advantage over firms that maximize profit.[3]

According to managerial theories of the firm, particularly the one developed by Baumol, the empirical conclusion is that the sales and profits of firms should be negatively correlated. If at least one empirical paper argued that there is no such correlation, then Baumol's theory would be disconfirmed.[4] It turns out that such papers do exist.

Mabry and Siders (1967) studied the correlation coefficients between sales and profits over twelve years (1952–1963) for 120 large American corporations. They demonstrated that there are positive significant correlations between sales revenue and profits. There were 25 firms for which the sales exceeded the level corresponding to the maximum profit, and, in most of the cases, there was a positive correlation between sales and profits. Mabry and Siders (1967, 371) concluded that: "The two hypotheses of sales maximization and profit maximization are not inconsistent."

Hall (1967) performed a multiple regression analysis for a sample of 99 American corporations selected from the list of major American corporations published annually by *Fortune*, broken down into 14 industries. Using data for the years 1960–1962, he tested the hypothesis that, if a company achieves a profit that exceeds the minimum, it increases its sales. He reached the conclusion that there are insignificant correlations between the independent variables and the dependent variable. Based on the above, he concluded that: "In general our findings lend no support to the SRM [sales revenue maximization – author's note] thesis" (Hall 1967, 154). Both empirical papers disconfirmed the hypothesis of sales revenue maximization with the profit constraint.

The profit-maximizing versus sales-maximizing strategy of the firm still lacks a conclusive theoretical and empirical solution. In other words, this is still an open question that encourages further research.

The main aim of this paper is to develop a model in which there is no discrepancy between the maximization of sales and the maximization of profit and to investigate its mathematical and economic properties. The assumptions of the model are the following:

– The objective of the firm is to maximize profit[5] by

$$\frac{\Delta P_t}{P_{t-1}} \geq \frac{\Delta P_{t-1}}{P_{t-2}} \tag{1}$$

where:

[3] Maybe the fact that American corporations lost to Japanese firms at some point was not because, unlike Japanese companies, American corporations maximized profit but because they were less efficient in maximizing sales.

[4] Lakatos (1999) stated that all hypotheses and theories are floating in a sea of anomalies. What protects them from being falsified is a protective belt of auxiliary hypotheses. Ascertained facts that are inconsistent with a theory or a hypothesis can falsify it only after a better theory has emerged. Therefore, the term disconfirmation will be used instead of the term falsification in this paper.

[5] This is operating profit (EBIT) calculated on a variable costing basis.

ΔP_t – the profit growth in period *t+1*,

P_{t-1} – the profit in period *t–1*,

that is, the profit growth rate in the current period should not be lower than the profit growth rate in the previous period. I assume the least restrictive version of (1), specifically that the profit growth rate in each period should be the same as in the previous period.[6]

– The sales revenue is linear functions of output;
– The total cost is linear functions of output;
– The costs can be divided into their fixed and variable elements;
– The firms produce single product or in multiproduct firms, the sales mix is constant;
– The unit variable costs are constant.

Staehle (1942) stated that many authors determined based on statistical investigations that marginal costs were constant in the observed range. Scherer (2001) presented the history of applications of the average costs depending on volume concept in the mainstream of economic theory. The assumption that the unit variable cost is constant has been confirmed by numerous empirical studies, among others those by Dean (1948), Johnston (1960, 1972), Walters (1963) and Koot and Walker (2001). Johnston (1960, 168) presents the final comment on his survey of cost functions that: "Two major impressions, however, stand up clearly. The first is that the various short-run studies more often than not indicate the constant marginal cost and the declining average cost as the pattern that best seems to describe the data that have been analyzed ".

Based on the study conducted by Blinder et al. (1998), it may be concluded that the marginal costs curve is U-shaped only in the case of 11% of firms in the USA. These studies point to the assumption that variable unit costs are constant and overlap with marginal costs.

Goldratt, the author of the theory of constraints, thought that the totally variable costs are the costs of direct materials (Goldratt and Cox 1984; Goldratt 1990a, 1990b; Noreen, Smith, and Mackey 1995). This is an extreme version of variable costing; however, in a firm that is completely automated and robotized (a so-called factory without light), the only variable costs are the costs of direct materials.

– The prices are constant.

Blinder et al. (1998) discussed 19 theories attempting to explain price stickiness in today's economy. They can be divided into theories based on the nature of costs, the nature of demand, the nature of contracts, the nature of market interactions, and imperfect information.

Kalecki (1942) thought that the rigidity of prices – determined with the use of the percentage gross margin – was due to monopolization of the market. He therefore argued that prices on industrial markets are set based on the costs and not on the demand. His reasoning followed from the assumption that a short-term marginal cost is constant and equal to an average variable cost (Kalecki 1972). Hicks (1972) made a distinction between fixed-price markets (industrial markets) and flexible-price markets (agriculture products and raw materials markets).

In the paper, I refer to Kalecki's views, modifying them by introducing the contribution margin rate expressed by the following formula:

$$s = \frac{p - k_v}{p}$$

which can be used to calculate the price:

[6] Information on the profit growth rate in a particular period is one of the most important indicators affecting changes in a firm's share prices.

$$p = \frac{1}{1-s} k_v$$

where:
s – the planned contribution margin rate;

p – the price;

k_v – the variable unit costs;

$\frac{1}{1-s}$ – the percentage mark-up of the contribution margin on the unit variable costs.

The price is also constant for constant variable unit costs and a constant mark-up on variable unit costs determined by a contribution margin rate planned by an oligopolistic firm.

– The production and sales are equal to demand; consequently, there are no stocks;

– There are no limitations in terms of demand in a particular period for profit maximization (1);

– There are no limitations in terms of production capabilities in a particular period for profit maximization (1);

– The initial sales are higher than the sales at the break-even point (BEP in short).

The sales revenue time paths, profit paths, sales growth rates, profit growth rates, and returns on sales will be defined in the contemporary[7] general model with the use of a non-homogeneous first-order difference equation derived from the formula for the degree of operating leverage (in short DOL). The properties of these paths not only show that the maximization of sales is consistent with the maximization of profit but also enable novel theoretical facts to be found. All the amounts in the numerical examples are in USD.

Another aim of this paper is to define the sensitivity of the sales growth rate and profit growth rate to a decrease in the initial sales and changes in the variable unit costs and fixed costs. The last goal of this papier is to investigate the consequences of a situation in which there are, on the one hand, binding constraints on the demand side or production capacity and, on the other hand, companies striving to maximize their profit (1).

This paper is organized as follows. Section II sets up the general model of growth of an oligopolistic firm. Section III presents the sensitivity of the results to alternative parametrizations of initial sales, unit variable costs, and fixed costs. Section IV cancels the two assumptions that there are no limitations in terms of demand and production capabilities in a particular period for profit maximization (1) and examines the consequences of binding constraints that relate to the demand or production capacity. Section V contains the conclusions. Appendix 1 presents the solution of the first-order non-homogeneous difference equation (8). Appendix 2 includes the data necessary to draw figure 1. Appendix 3 includes the data necessary to draw figure 2. Appendix 4 includes data necessary to compile table 1 and the data necessary to draw figure 3. Appendix 5 includes the data necessary to compile table 2 and to draw figure 4. Appendix 6 includes the data necessary to draw figures 5 and 6. Appendix 7 shows the consequences of reducing fixed costs by 25%. Appendix 8 presents the results of decrease in variable unit costs by 25% and indifferent increase in fixed costs.

II. General firm growth model

A. Necessary sales time path

[7] I call this model contemporary because the assumption that the total costs and total sales revenue are linear functions of output approximately corresponds to the contemporary conditions in large oligopolistic corporations.

It follows from the DOL formula that the profit growth rate is equal to the product of the DOL and the sales growth rate:

$$d_{pt+1} = d_{ot} d_{st+1} \tag{2}$$

The sales growth rate that is necessary to achieve the planned profit growth rate is equal to

$$d_{st+1} = \frac{d_{pt+1}}{d_{ot}} \tag{3}$$

where:

d_{pt+1} – the planned or achieved profit growth rate in period *t+1*,

d_{ot} – The DOL in period *t*,

d_{st+1} – the sales growth rate in period *t+1*,

which is the hidden model of firm growth. To demonstrate this, the DOL will be expressed as the relation between the contribution margin and the profit:

$$d_{ot} = \frac{d_{pt+1}}{d_{st+1}} = \frac{\dfrac{d_{st+1} M_t}{P_t}}{\dfrac{d_{st+1} S_t}{S_t}} = \frac{M_t}{P_t} \tag{4}$$

because the profit growth is

$$\Delta P_{t+1} = P_{t+1} - P_t = (1 + d_{st+1}) S_t - (1 + d_{st+1}) K_{vt} - K_{ft+1} - S_t + K_{vt} + K_{ft} =$$

$$= d_{st+1}(S_t - K_v) = d_{st+1} M_t \tag{5}$$

Formula (3) may be transformed as follows:

$$\frac{S_{t+1} - S_t}{S_t} = \frac{d_{pt+1}}{d_{ot}} \tag{6}$$

allowing the determination of the sales growth given the assumption that an enterprise wants to achieve the profit growth rate from the initial period:

$$S_{t+1} - S_t = \frac{d_p}{d_{ot}} S_t = \frac{d_p}{\dfrac{M_{wt}}{P_t}} S_t = \frac{d_p}{\dfrac{sS_t}{sS_t - K_f}} S_t = \frac{d_p}{s}(sS_t - K_f) = d_p S_t - \frac{d_p}{s} K_f \tag{7}$$

Finally, a non-homogeneous difference equation of the first degree is derived from (7):

$$S_{t+1} - (1 + d_p) S_t = -\frac{d_p}{s} K_f \tag{8}$$

where:

S_t – the sales in period *t* necessary to achieve initial profit rate growth d_p in the same period,

M_t – the contribution margin in period *t*,

$M_t = S_t - K_{vt}$

P_t – the profit in period *t*,

ΔP_{t+1} – the profit growth in period *t+1*,

K_{vt} – the total variable costs in period *t*,

K_f – the total fixed costs in period *t*.

The definite discrete solution of the first-order non-homogeneous difference equation[8] (8) is

$$S_t = M_s(1+d_z)^t + S_{BEP} \qquad (9)$$

where:

S_{BEP} – the sales for the break-even point (in short BEP),

$$S_{BEP} = \frac{K_f}{s}$$

s – the initial contribution margin rate,

$$s = \frac{M_0}{S_0}$$

S_0 – the initial sales,

M_s – the initial safety margin,

$M_s = S_0 - S_{BEP}$

The expression $M_s(1+d_p)^t$ represents a deviation from a specific intermediate equilibrium position, in this case M_s. If I add a certain element, in this case S_{BEP}, the time path will shift upwards, resulting in a change in the level of equilibrium. In other words, there will be a change in the level at which the convergence or divergence of the time series is measured.

To establish the necessary sales growth rate (in short NSGR), I use (9) first to determine the sales growth:

$$\Delta S_{t+1} = M_s(1+d_p)^{t+1} + S_{BEP} - M_s(1+d_p)^t - S_{BEP} =$$

$$= M_s(1+d_p)^t(1+d_p-1) = d_p M_s(1+d_p)^t \qquad (10)$$

[8] The solution of the first-order non-homogeneous difference equation (8) is presented in appendix 1.

Then the NSGR to achieve the initial profit growth rate in the following periods is

$$d_{st+1} = \frac{\Delta S_{t+1}}{S_t} = \frac{d_p M_s (1+d_p)^t}{M_s (1+d_p)^t + S_{BEP}} = \frac{d_p}{1+\dfrac{S_{BEP}}{M_s (1+d_p)^t}} = \frac{d_p}{1+\dfrac{1}{m_e (1+d_p)^t}} = \frac{d_p}{1+\dfrac{1}{m_s r_0 (1+d_p)^t}}$$

$$d_{st+1} = \frac{d_p}{1+\dfrac{1}{m_s r_0 (1+d_p)^t}} \quad (11)$$

The amount of profit in particular periods is calculated as follows:

$$P_t = sS_t - K_f = s[M_s (1+d_p)^t + S_{BEP}] - K_f \quad (12)$$

which constitutes the basis for determining the return on sales:

$$ROS_t = \frac{P_t}{S_t} = \frac{s[M_s (1+d_p)^t + S_{BEP}] - K_f}{M_s (1+d_p)^t + S_{BEP}} = \frac{s}{1+\dfrac{1}{m_e (1+d_p)^t}} = \frac{s}{1+\dfrac{1}{m_s r_0 (1+d_p)^t}} \quad \mathbf{(13)}$$

where:

ROS_t – the return on sales in period *t*,

m_e – the initial sales surplus rate over the break-even point,

$$m_e = \frac{S_0 - S_{BEP}}{S_{BEP}} \quad (14)$$

m_s – the safety margin rate

$$m_s = \frac{S_0 - S_{BEP}}{S_0} \quad (15)$$

r_0 – the relation of initial sales to sales for the break-even point,

$$r_0 = \frac{S_0}{S_{BEP}} \quad (16)$$

B. Convergent time path

For the time path to be convergent, $M_s (1+d_p)^t$ should tend to zero when *t* tends to infinity. For this to happen, the profit growth rate must be negative (which does not mean a loss), that is:

If $d_p < 0$, then

$$\lim_{t \to +\infty} M_s (1+d_p)^t = 0 \quad (17)$$

Therefore

$$\lim_{t\to\infty} S_t = S_{BEP} \tag{18}$$

that is, the sales asymptotically tend to the break-even point (in short BEP) and therefore

$$\lim_{t\to\infty} P_t = sS_{BEP} - K_f = M_{wBEP} - K_f = 0 \tag{19}$$

which means that the profit asymptotically tends to zero, because, for the break-even point, the contribution margin is equal to the fixed costs, and the return on sales (in short ROS) tends to zero:

$$\lim_{t\to\infty} ROS_t = \lim_{t\to\infty} \frac{s}{1+\frac{1}{m_s r_0 (1+d_p)^t}} = 0 \tag{20}$$

I can point to two cases of a negative profit growth rate. The first applies to a constant demand in the branch and inflow of new capital. This situation is similar to the inflow of capital in the model of perfect competition. A negative profit growth rate depends on the pace of the additional capital inflow. This may be the case in an oligopoly, in which the price competition is excluded and, as a result of the capital inflow and the increase in supply, a drop in sales in an existing firm is the only reaction. If a one-time capital inflow is sufficient, the profit growth rate is -100% and the BEP is achieved within one period. The second example of a convergent path may concern a traditional branch of manufacturing in which the demand for its products decreases from period to period and therefore the sales tend to the BEP and the profit and ROS asymptotically tend to zero.

C. Divergent time path

Since the main aim of this paper is to explain the contemporary maximization of sales, further considerations will focus on the divergent sales path and its consequences. This path occurs when $M_s(1+d_z)^t$ tends to +∞ when t tends to +∞. The stationary intermediate equilibrium position for t = 0:

$$R = M_s(1+d_p)^0 + S_{BEP} = S_0 - S_{BEP} + S_{BEP} = S_0 \tag{21}$$

If the profit growth rate d_p is positive, then

$$\lim_{t\to+\infty} M_s(1+d_z)^t = +\infty \tag{22}$$

Therefore,

$$\lim_{t\to+\infty} S_t = +\infty \tag{23}$$

This does not mean, however, that the NSGR tends to +∞. Using (22), I can define the limit to which it tends.

$$\lim_{t \to +\infty} d_{st} = \lim_{t \to +\infty} \frac{d_p}{1 + \frac{1}{m_s r_0 (1 + d_p)^t}} = d_p \quad (24)$$

It turns out that the NSGR tends asymptotically to the initial profit growth rate.

The conclusion from (22) is that the profit asymptotically tends to +∞.

$$\lim_{t \to \infty} P_t = sS_t - K_s = s[M_s (1 + d_z)^t + S_{BEP}] - K_s = +\infty \quad (25)$$

This does not mean that the ROS also tends to +∞.

$$\lim_{t \to \infty} ROS_t = \lim_{t \to \infty} \frac{s}{1 + \frac{1}{m_e (1 + d_p)^t}} = s \quad (26)$$

According to the result in (26), the ROS asymptotically tends to the initial contribution margin rate.

Dynamic instability of the equilibrium is observed for a positive profit growth rate. Figure 1 illustrates the sales time paths and profit path.

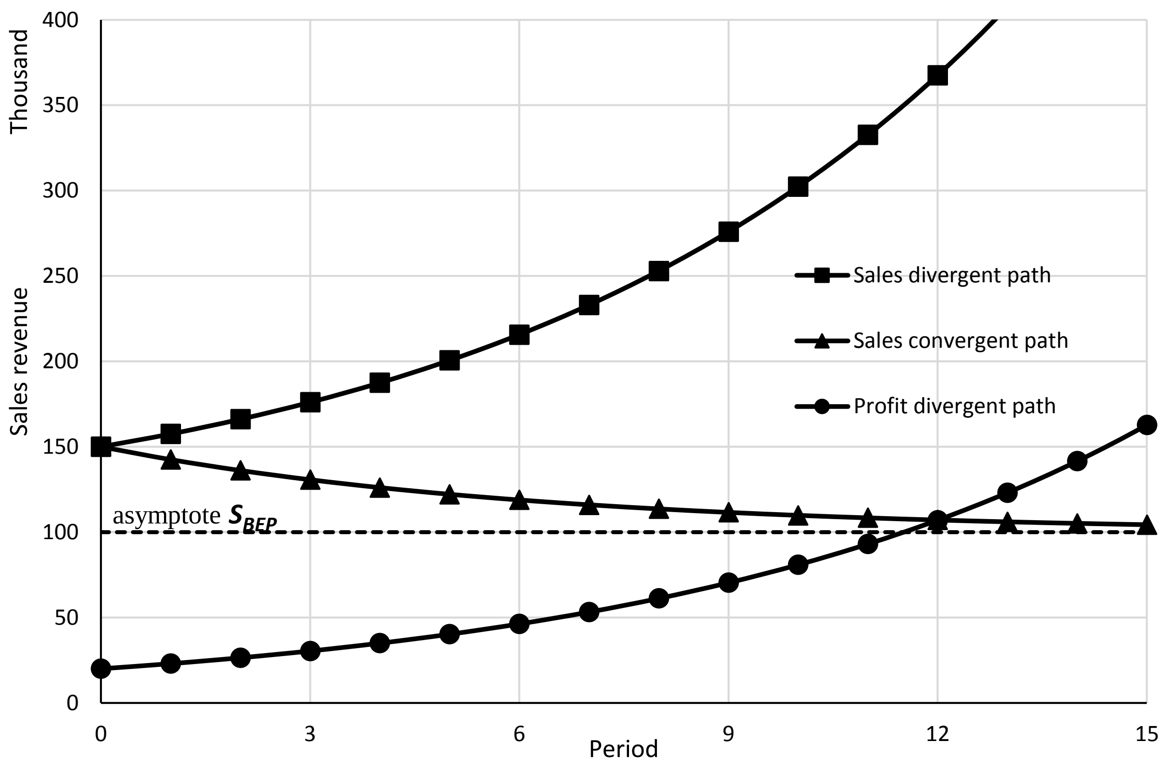

FIG. 1.—Divergent sales and profit time path and convergent sales time path

Source: Own work. The data necessary to draw figure 1 are included in appendix 2.

The analyzed divergent time path meets the following condition:

$$S_0 > S_{BEP} \quad (27)$$

that is, the firm is in the profit zone and the previously achieved profit growth rate is positive d_p. The profit time path is also divergent.

How will the sales growth rate react when an enterprise tries to maintain its previously achieved profit growth rate? The dependency between the sales growth rate and the time will be analyzed to answer this question. The NSGR formula (11) means that, to maintain the once-achieved profit, the growth rate increases in subsequent periods. For a positive profit growth rate, the fraction in the denominator in (11) will decrease with an increase in *t*, which means that the entire denominator will decrease, thereby causing an increase in the sales growth rate in the following periods. The initially achieved profit growth rate that a company aims to maintain in the future becomes an asymptote, and the NSGR approaches it from the bottom. Two regularities that are required to achieve the initial profit growth rate in the following periods are that the NSGR increases and that the NSGR tends to the upper asymptote, which is the initial growth rate of profit; these may be considered as two novel theoretical facts found with the use of the general firm growth model.[9]

Figure 2 presents the NSGR and ROS for the divergent sales path.

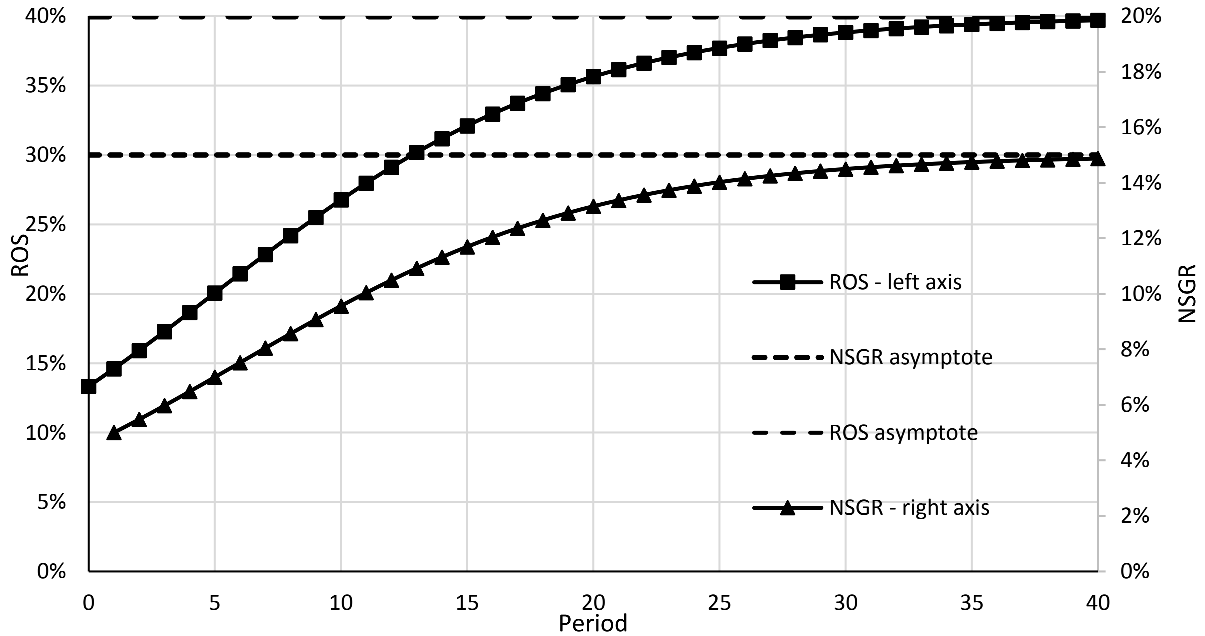

FIG. 2.—NSGR and ROS for the divergent sales time path

Source: Own work. The data necessary to draw figure 2 are included in appendix 3.

The NSGR and ROS in figure 2 are non-linear rising functions. The upper asymptote of the NSGR for the divergent sales path is determined by the initial profit growth rate. It should be pointed out that this occurs despite the sales path being divergent. It is demonstrated that in following periods an increasingly high sales growth rate is necessary to maintain the initially achieved profit growth rate.

The limit to which the ROS tends, that is, the relation between profit and sales, is not an indeterminate value, although both profit and sales tend to +∞. The upper asymptote is the

[9] Lakatos (1999, 5) claimed that: "Thus, in a progressive research programme, theory leads to the discovery of hitherto unknown novel facts" and (1999, 33) "Let us say that such a series of theories is theoretically progressive (or 'constitutes a theoretically progressive problem shift') if each new theory has some excess empirical content over its predecessor, that is, if it predicts some novel, hitherto unexpected facts."

contribution margin rate. This is another novel theoretical fact found with the use of the general model.

I will explain why maintaining the initial profit growth rate requires an increasingly high sales growth rate in subsequent periods with reference to the DOL. After each increase in sales, there is a decrease in the DOL. This means that achieving the same profit growth rate in the next period, according to (3), requires an increase in the sales growth rate. Figure 3 presents the cause–effect time sequence between the NSGR and the DOL.

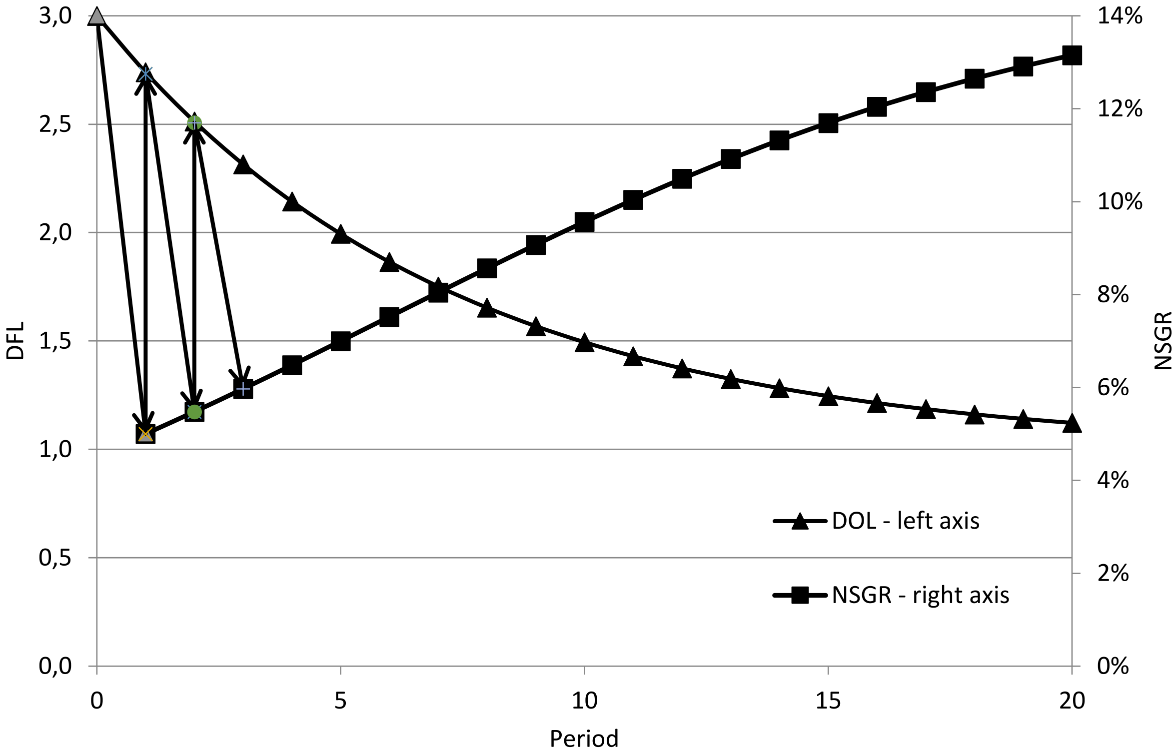

FIG. 3.—Cause–effect time sequence for the first three periods
Source: Own work. The data necessary to draw figure 3 are included in appendix 4.

The cause–effect time sequence presented in figure 3 is explained by the cause–effect time sequence for the first three periods presented in table 1.

TABLE 1

CAUSE–EFFECT TIME SEQUENCE FOR THE FIRST THREE PERIODS

Period	DOL	NSGR
0	3.0000	
1	2.7391	5.00%
2	2.5123	5.48%
3		5.97%

Source: Own work. Appendix 4 includes the data necessary to compile table 1.

The basic conclusion from the analysis of the dependency between the DOL and the NSGR is that the lower the DOL, the higher the NSGR, enabling the initially achieved profit growth rate to be maintained in the following periods. Table 1 illustrates this regularity. The DOL in the initial period is 3. As a result, according to (11) in the first period, the NSGR should have been 5% to achieve the initial 15% profit growth rate. I will attain the same result using modified (2), which considers the profit growth rate in the first period as a constant

$$d_{st+1} = \frac{d_p}{d_{ot}} \qquad (28)$$

The NSGR in the first period causes a decrease in the DOL to 2.74 in the same period, according to the DOL formula as a function of the sales growth rate (Mielcarek 2006, 204):

$$d_{ot} = \frac{(1+d_{st})d_{ot-1}}{1+d_{st}d_{ot-1}} \qquad (29)$$

The decrease in the DOL to 2.74 in the first period causes an increase in the NSGR in the second period to 5.48% to maintain the profit growth rate achieved in the first period. This in turn causes a drop in the DOL in the second period to 2.51, which results in an increase in the NSGR in the third period to 5.97% and so on.
Equality (11) and (28)

$$\frac{d_p}{d_{ot}} = \frac{d_p}{1+\frac{1}{m_s r_0 (1+d_p)^t}} \qquad (30)$$

implies that DOL is a discrete function of time

$$d_{ot} = 1+\frac{1}{m_s r_0 (1+d_p)^t} \qquad (31)$$

The advantage (31) over (29) is that with (31) I can immediately define the DOL value for any period because (31) follows from the general solution of difference equation (8). If I use (29) and (28), then DOL and NSGR calculations are required for all preceding periods. This means that I use the iterative method to determine the DOL and NSGR time paths. However, if I want to analyze the cause-and-effect time sequence between the DOL and NSGR, then (28) and (29) are more useful.[10] They present the change patters of the DOL and NSGR between two consecutive periods and they show the direct mutual impact of the DOL and NSGR.

Using (31) allows to easily determine the DOL asymptote (limit). In the case of the divergent sales function

$$\lim_{t\to\infty} d_{ot} = 1 \qquad (32)$$

that is, the number one constitutes the asymptote of the DOL function.

III. Sensitivity analysis

The sensitivity analysis will concern the impact of a decrease in initial sales, that is, the safety margin, changes in the variable unit costs, and changes in the total fixed costs on the necessary sales paths and NSGR.

[10] The operation of feedback in this model is demonstrated. There are two types of feedback - positive, increasing the NSGD and negative, reducing the DOL. The larger the NSGR, the smaller the DOL and consequently the smaller the DOL, the greater the NSGR.

A. Economic recovery

The dependency described in Section 2, namely that maintaining the initially achieved profit growth rate requires increasingly high sales growth rates, is of particular importance for the period when an economic recovery starts. When an economy is emerging from a recession, the safety margin and safety margin rate are at a low level.[11] This means that the DOL is very high. In such a situation, according to (2), only a slight sales increase is necessary to generate a high profit growth rate.

The sensitivity analysis at this point will consist of analyzing the impact of the decreasing safety margin rate on the NSGR. Table 2 presents a comparison of a numerical example for the general model, in which the safety margin rate is 33.33%, and an example for the period of emergence from a recession, when the safety margin rate is 10%.

TABLE 2

NECESSARY SALES GROWTH RATES FOR THE GENERAL MODEL AND FOR EMERGING FROM A RECESSION

Period	NSGR: general model	DOL: general model	NSGR: recovery	DOL: recovery
0		3.00		10.00
1	5.00%	2.74	3.00%	7.92
2	5.48%	2.51	3.79%	6.33
3	5.97%	2.32	4.74%	5.10
4	6.48%	2.14	5.89%	4.15
5	7.00%	1.99	7.23%	3.42
6	7.52%	1.86	8.76%	2.86
7	8.04%	1.75	10.47%	2.43
8	8.56%	1.65	12.32%	2.10
9	9.07%	1.57	14.26%	1.85
10	9.56%	1.49	16.23%	1.65

Source: Own work. Appendix 4 includes the data necessary to compile table 2.

The regularities in table 2 are more indicative when presented graphically. Figure 4 provides the NSGR.

[11] The assumption is that a company generates small profits in this period.

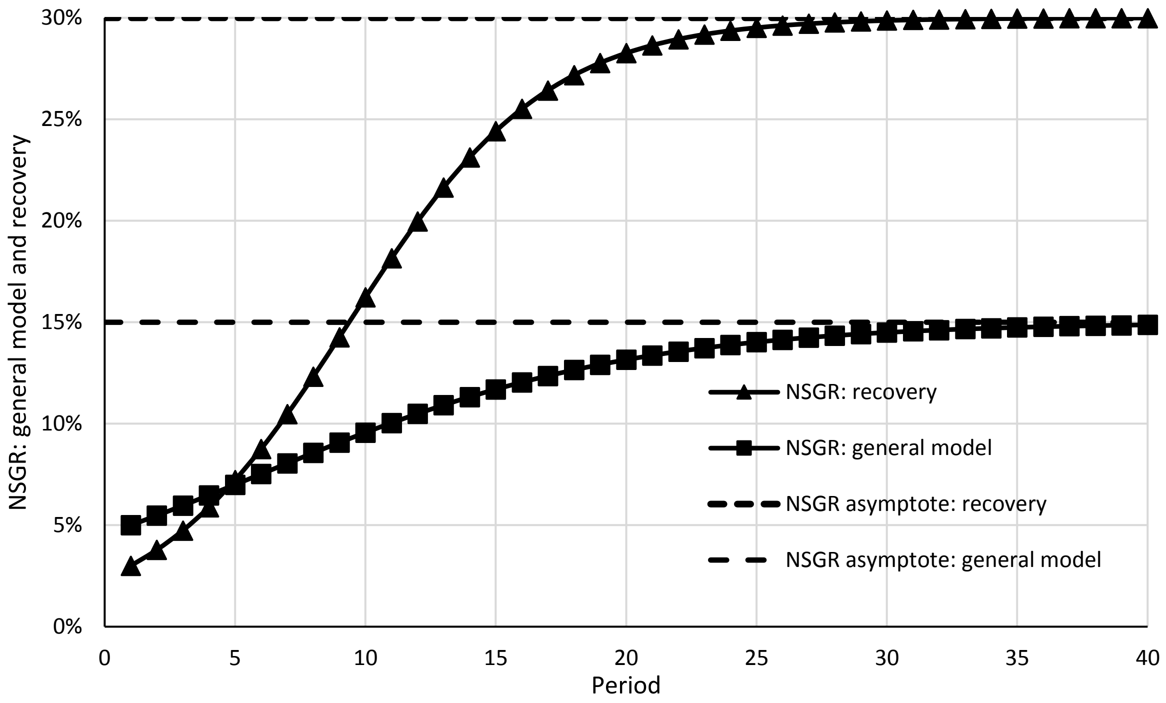

Fig. 4.—Necessary sales growth rates for the general model and recovery
Source: Own work. The data necessary to draw figure 4 are included in appendix 5.

A 30% profit growth rate in the initial period at the stage of emerging from a recession is easy to achieve when the DOL is 10, because it requires a necessary increase in sales of only 3%. Unfortunately, in the following periods, the NSGR increases rapidly, exceeding the NSGR from the general model as early as in the fifth period, when the profit growth rate to be maintained is 15%. This points to the hypothesis that, as a result, at the end of the recovery period, firms cannot achieve an increasing NSGR because it is too high; what is more, the growing NSRG is calculated from the ever-higher sales base. These dependencies cause the issue that the real profit growth rate should be falling. The first conclusion is that, at the beginning of the recovery period, companies should limit the initial sales growth rate to prevent the NSGR from reaching an excessive level that will be impossible to maintain in the future.

Such measures are hindered by competition. Heuristic metaphors of competition will help to explain this phenomenon.[12] Robinson (1971, 101) stated: “Thus each has to run to keep up with the rest.” Prince, the former head of Citigroup, said the following about one month before the financial crisis broke out: “But as long as music is playing, you’ve got to get up and dance. We’re still dancing.” (Blinder, 2013: Kindle Location 125). These metaphors can be modified based on the general model of growth. The first can be expressed as follows: “Thus each has to run faster and faster to keep up with the increasing pace of the rest.” After modification, the second metaphor will read as follows: “But as long as music is playing faster and faster, you’ve got to get up and dance faster and faster.” Unfortunately, when someone has to run or dance faster and faster, he will eventually lose his breath or, in the worst-case scenario, have a heart attack.[13] However, if a company did not act in this way from the beginning of recovery, it would lose its market share, which would be unacceptable for its shareholders and managers.

B. *Impact of a decrease in variable unit costs*

[12] Kuhn (1996) included ontological and heuristic metaphors for the paradigm.
[13] The 2007–2009 global financial crisis can serve as an example.

It can be assumed that it would be easier for a company to maintain the initial profit growth rate if the variable unit costs were lowered. In other words, this would be a solution for lowering the NSGR in the following periods; that is, it would facilitate the achievement of the company's profit maximization goal.

To examine the impact of a decrease in the variable unit costs on the NSGR, the NSGR before and after such a decrease will be compared. A hypothesis contrary to the one referred to above, that is, the NSGR increases instead of decreases with a decrease in the variable unit costs, means that the situation becomes more challenging in terms of achieving the enterprise's objective consisting of maximizing the profit:

$$d_{st+1c} > d_{st+1} \quad (29)$$

after substituting into the inequality (11)

$$\frac{d_p}{1+\frac{1}{m_{sc} r_{0c} (1+d_p)^t}} > \frac{d_p}{1+\frac{1}{m_s r_0 (1+d_p)^t}} \quad (30)$$

After the relevant multiplications and divisions, the inequality is:

$$1+\frac{1}{m_s r_0 (1+d_p)^t} > 1+\frac{1}{m_{sc} r_{0c} (1+d_p)^t} \quad (31)$$

and after simplifications, multiplications, and divisions I obtain:

$$m_{sc} r_{0c} > m_s r_0 \quad (32)$$

That is, after a decrease in the variable unit costs, the NSGR needed to maintain the previous profit growth rate will increase if the safety margin rate, given a decrease in the variable unit costs, is higher than in the initial conditions, which means after substituting (15) that:

$$\frac{S_0}{S_{BEPc}} - 1 > \frac{S_0}{S_{BEP}} - 1 \quad (33)$$

and after simple transformations I obtain:

$$S_{BEP} > S_{BEPc} \quad (34)$$

that is,

$$\frac{K_f}{s} > \frac{K_f}{s_c} \quad (35)$$

which means that

$$s_c > s \quad (36)$$

that is,

$$\frac{S_t - K_{vtc}}{S_t} > \frac{S_t - K_{vt}}{S_t} \quad (37)$$

and after transformations

$$K_{vt} > K_{vtc} \quad (38)$$

where:

d_{st+1c} – the changed NSGR in period *t+1*, caused by the diminishing of the unit variable costs,

m_{sc} – the changed safety margin rate, caused by the diminishing of the unit variable costs,

r_{0c} - the changed initial sales to BEP relation, caused by the diminishing of the unit variable costs,

s_c – the changed contribution margin rate, caused by the diminishing of the unit variable costs,

S_{BEPc} – the sales for the BEP, changed as a result of a decrease in the variable unit costs,

K_{vtc} – the total variable costs, changed as a result of a decrease in the variable unit costs in period *t*.

Condition (38) is met, because the assumption was to analyze the impact of a decrease in the variable costs compared with the initial conditions. It was therefore possible to demonstrate that, if the variable unit costs decrease, then the NSGR will increase and it will be more difficult to achieve identical profit growth rates in subsequent periods. A decrease in the variable unit costs does not improve a company's situation: quite the contrary, it deteriorates, because, after a decrease in the variable unit costs, the sales and the necessary sales growth rate move to a higher path. These are other novel theoretical facts discovered with the use of sensitivity analysis.

I consider a situation in which the variable unit costs decrease by 25% in the sixth period. Figure 5 illustrates the outcome of this change.

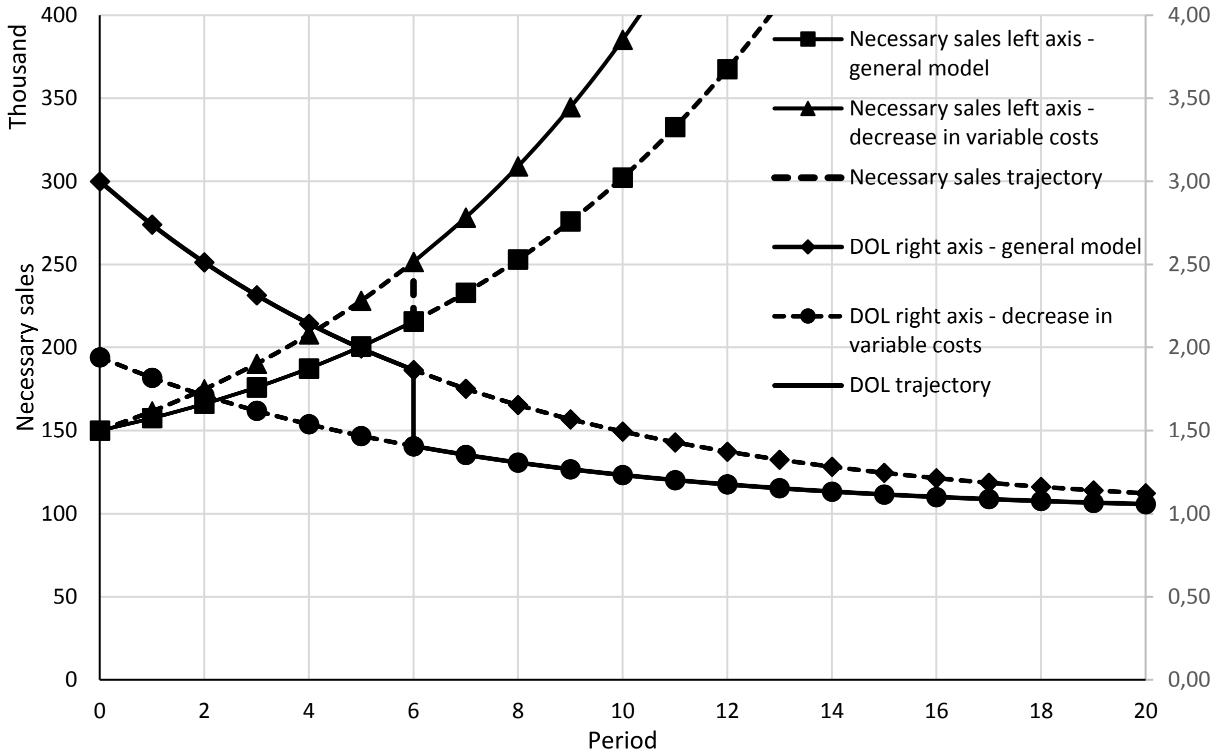

FIG. 5.—Change in the divergent time path of necessary sales and DOL after a 25% decrease in the unit variable costs

Source: Own work. The data necessary to draw figure 5 are included in appendix 6.

For the divergent necessary sales time path in the sixth period, a discontinuity point occurs and the sales point moves to the new upper path. This jump is caused by a downward shift in the DOL function due to a 25% decrease in the unit variable costs. Consequently, a DOL point jump is made to the lower DOL function that plots the trajectory of this motion. In other words, the fall in the unit variable costs causes the DOL function to move downwards, and this causes the necessary sales function to move upwards. A given drop in the unit variable costs determines the necessary sales function and DOL function from their families to which respectively the points of necessary sales and DOL carry out the jumps. Achieving the previous profit growth rate becomes increasingly difficult, because it requires significantly higher sales values than the initial conditions.

Figure 6 presents the changes in the NSGR for a decrease in the variable unit costs.

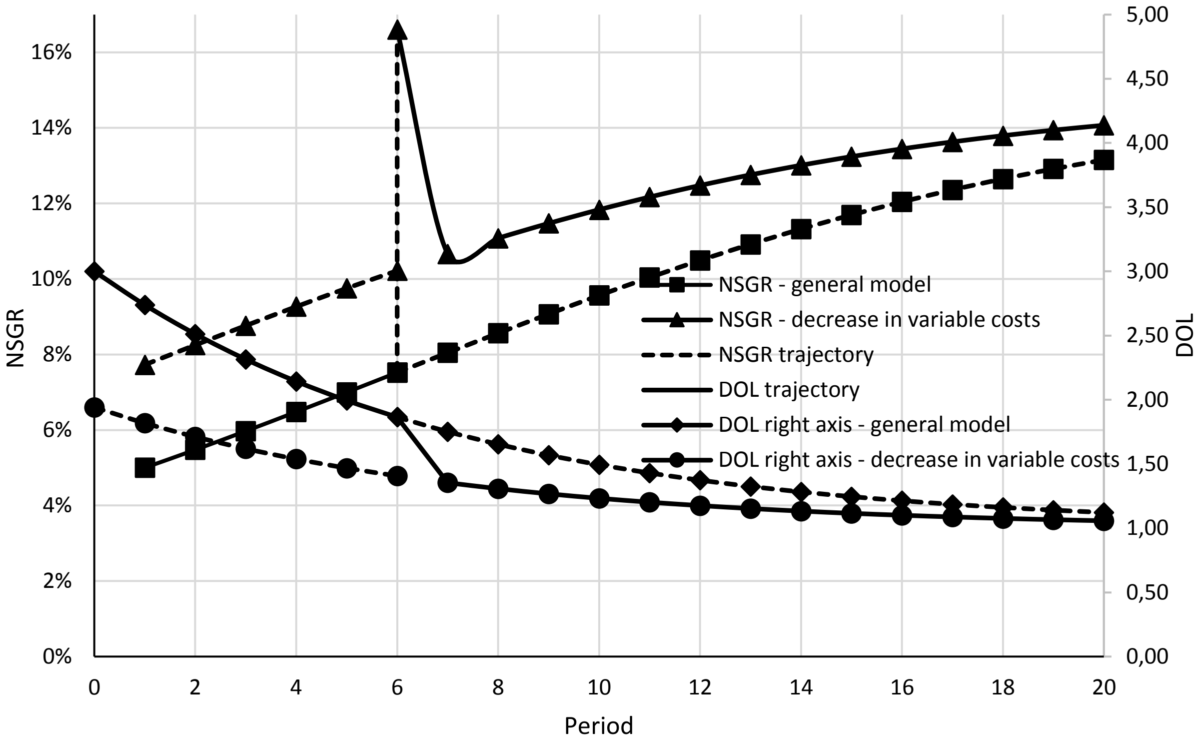

FIG. 6.—Changing the time path of the NSGR for a divergent sales time path after a 25% decrease in the unit variable costs

Source: Own work. The data necessary to draw figure 6 are included in appendix 6.

There is a discontinuity point in the sixth period. Since the necessary sales point moves to the upper necessary sales function, as shown in figure 5, adjusting to the new conditions defined by the lower variable unit costs requires a one-time dislocation of the NSGR point, which lies outside the new upper NSGR function. In the next step, the NSGR point returns to the new function, in which the subsequent NSGR is higher than that resulting from the initial conditions. In period 6, a one-time unique NSRD is not defined by a DOL point on a new lower-located function, so the DOL point jumps from the upper function to the lower function in period 7. Instead of facilitating the achievement of the company goals, the decrease in the variable unit costs makes it more difficult.

Referring to figure 3, which presents the cause–effect time sequence between the NSGR and the DOL, figures 5 and 6 can be interpreted as well. As a result of an increase in the contribution margin rate caused by a decrease in the variable unit costs, the DOL function moves downwards. In particular periods, the DOL will be lower than the values on the initial conditions path. For a particular DOL, achieving the initial profit growth rate in the following period will require a higher NSGR, which means that this point is located on the NSGR function moved upwards. As a result, according to (28), the DOL in this period will decrease and so on.

C. Indifferent decrease in the variable unit costs and increase in the total fixed costs

The impact of the decrease in the fixed costs on the changes in the necessary sales and NSGR for the same profit growth rate in subsequent periods I will also analyze. If the fixed costs decrease in a particular period, the NSGR will increase; therefore, as in the case of the decrease in variable costs, it will hinder the achievement of the company's objective instead of facilitating it.[14] The divergent sales path rises with the discontinuity point in the period when

[14] The explanation for this regularity is provided in appendix 7.

the decrease in the fixed costs takes place. This is another novel theoretical fact found through the sensitivity analysis.

The dislocation of the sales point to the new divergent necessary sales path, which moves upwards, requires a one-off jump of the NSGR, which lies beyond the new upper divergent sales growth rate path. In the following period, the NSGR returns to the new convergent path, along which the subsequent NSGR is higher than that resulting from the initial conditions.

There may also be a case in which the variable unit costs decrease, and the fixed costs increase simultaneously. This gives rise to the problem of the unchanged path of the NSGR, which can be formulated as follows: what should the fixed costs growth rate be for a particular decrease in the variable unit costs for the sales time path and for the NSGR time path to remain unchanged?

After a decrease in the variable unit costs, the increase in the fixed costs should make the NSGR before and after the changes (11) equal:

$$\frac{d_p}{1+\frac{1}{m_{sc}r_{0c}(1+d_p)^t}}=\frac{d_p}{1+\frac{1}{m_s r_0(1+d_p)^t}} \qquad (39)$$

These will be equal when

$$m_{sc}r_{0c}=m_s r_0 \qquad (40)$$

After substituting (38) with (15):

$$\frac{S_{BEPc}}{S_0}-1=\frac{S_{BEP}}{S_0}-1 \qquad (41)$$

Therefore,

$$S_{BEPc}=S_{BEP} \qquad (42)$$

that is, substituting the BEP formula for an increase in the fixed costs and a decrease in the variable unit costs on the left side of (42):

$$\frac{K_f(1+d_f)}{\frac{S-K_v(1+d_v)}{S}}=\frac{K_f}{s} \qquad (43)$$

The indifferent fixed costs growth rate for a particular rate of decrease in the variable unit costs is calculated as follows:

$$d_f=\frac{S-K_v(1+d_v)}{sS}-1=\frac{S-K_v(1+d_v)-S+K_v}{S-K_v}=-\frac{d_v}{S_t-K_v}K_z=-\frac{d_v}{\frac{S}{K_v}-1}=-\frac{d_v}{\frac{S}{S\ (1-s)}-1}$$

$$d_f=-\frac{1-s}{s}d_v \qquad (44)$$

where:

d_f – the indifferent fixed costs’ growth rate,

d_v – the initial variable unit costs’ growth rate.

The indifferent fixed costs' growth rate is determined by the contribution margin rate and the variable unit costs' growth rate. Given the assumption that the decrease in the variable unit costs is 25% and the contribution margin rate for the initial conditions is 40%, then the rate of increase in the fixed costs for the unchanged necessary sales path and the NSGR will be equal to

$$d_{ft+1} = -\frac{1-0.4}{0.4}(-0{,}25) = 0.375 \qquad (45)$$

The fixed costs' growth rate should be 37.5%. For this rate figures 5 and 6 are transformed into figures 1 and 2, respectively, for the divergent sales path and the NSGR[15].

In the case of mergers and acquisitions, it is often the case that the variable unit costs decrease and the fixed costs change. The following five cases may then occur for a particular decrease rate in the variable costs:

– if the fixed costs do not change, the NSGR point moves to the upper path;
– if the fixed costs decrease, the NSGR point moves to an even higher path;
– if the fixed costs' growth rate is lower than the indifferent rate (44), the NSHR point moves to the NSGR path located higher than the initial path but lower than the path for unchanged fixed costs;
– if the fixed costs' growth rate is equal to the indifferent rate (44), the NSGR point is located on the unchanged path; and
– if the fixed costs' growth rate is higher than the indifferent rate (44), the NSGR point moves to the path located below the initial path.

Cases one, two, and three make it more difficult for the management to maximize the profits after mergers and acquisitions, which may lead to disappointment with the results on the part of shareholders and the stock market. Cases four and five, paradoxically, facilitate profit maximization. The two last cases indicate that there is a discrepancy between changes in profit in the short term when a decrease in the variable costs and fixed costs requires an increase in the NSGR, thereby making it more difficult for the company to achieve its profit maximization goal. This contradiction is another novel theoretical fact found with the use of the sensitivity analysis.

Similar cases may be observed in innovative companies. The three first cases indicate that innovative companies and those that apply cost accounting and tools lowering the variable unit costs, such as Kaizen Costing and Target Costing, should be particularly aggressive on the domestic and international markets. Toyota is a good example of such behavior as a company that evolved from a small textile business into the biggest global car manufacturer.

IV. Binding constraints

The explanations and hypotheses concerning the functioning of today's economy are based on the findings in Sections II and III. I mentioned the metaphor according to which competition consists of dancing or running faster and faster, which, in the best-case scenario, may result in breathlessness. In other words, sooner or later a firm will encounter either a binding demand constraint or a binding production capacity constraint. This statement disregards two assumptions, namely that the firm does not encounter the demand constraint and that the firm does not encounter the production capacity constraint in particular periods. After such constraints occur, the management takes measures that can create the impression that it is no longer striving to achieve the objective of maximizing profit but instead replacing it with other objectives. However, with new or more serious obstacles hindering the achievement of the

[15] Appendix 8 presents these effects.

objective (1), the management concentrates on using the existing tools and finding new ones to achieve this objective.

The management develops a mentality that is functional towards achieving the firm's objective, that is, maximizing profit. If the managers start to think that their remuneration, importance, and prestige depend on sales, the sales growth rate, the share in the market or the number of employees, this is the functional element of their awareness in terms of profit maximization. Actions that are useful for the management actually contribute to profit maximization.

There are several such actions. First, attempting to overcome the demand constraint requires a change in the position of the consumer on the market. Achieving increasingly more sales is impossible on a market with consumer autonomy. The technostructure starts manipulating consumers (Galbraith 1967, 1973). Gaining control over consumers' preferences is one of the ways to overcome the constraints of the market. Consumers associate their well-being with possessing the newest goods. Needs are created, and marketing and advertisement are the tools used to manipulate consumers, hence the dynamic growth and increasing expenses incurred by firms in this respect. These measures undermine the notion of autonomous consumers who allocate resources to satisfy their needs, communicating with firms that maximize their profit through the market prices. Firms (technostructure) start to control their markets through the above actions.

The second measure, that is, shortening the life cycle of products, requires a high level of innovativeness and the release of new models or generations of products to replace the old ones. The third set of measures includes related and unrelated diversification. The first one is even applied in automotive companies that produce premium cars, which may seem like a surprising activity (e.g. Audi A1 or A-Class Mercedes). The differentiation of products is the fourth one. Expenditures on marketing and advertisement would not translate into the expected results if the products of companies from the same branch did not differ, hence the marketing principle: differentiate or die (Trout and Rivkin 2000). The fifth type of measure is the personalization and customization of products.

Mergers and acquisitions are the sixth way to maintain the necessary market expansion. A high propensity on the part of managers to take such actions is functional in terms of profit maximization. According to Section III, changes in the fixed costs are important for achieving the NSGR. In the case of a merger or acquisition of a firm, a decrease in the fixed costs compared with the total fixed costs before the merger is one of the indicators of success of such a transaction. If this is the case, there is a significant increase in the firm's profit. The price that needs to be paid for this success is a decrease in the DOL and an increase in the NSGR, which was explained in Section III and in Appendix 7.

Innovations, mergers, and acquisitions may lead to a decrease in the variable unit costs. Using methods such as Kaizen Costing or Target Costing for cost management may also lead to a decrease in the variable costs. According to Section III, a decrease in the variable unit costs translates into a paradoxical effect, specifically an increase in the profits in a particular period and an increase in the ROS, but, from a longer time perspective, problems arise in maintaining the previously achieved profit growth rate due to an increase in the NSGR.

The transformation of the market in particular industries is one of the main consequences of firms striving to maximize their profit. Binding demand constraints and reactions to them lead to the transformation of particular markets into oligopolistic ones. Referring to the notion of killer competition used by Trout and Rivkin (2000), it forces oligopolistic firms to strive to maximize their sales and market share.

An increase in sales, supported by the shaping of consumers' preferences, increased expenditures on promotion and advertisement, product innovations, the deepening and widening of the market, and the process of mergers and acquisitions on the domestic market may turn out to be insufficient to ensure the NSGR. It is then that international expansion becomes indispensable. It may therefore be stated that the described relation between the profit growth rate and the NSGR is one of the factors that also explain the globalization processes in the economy.

The accession of new member states in economic organizations is one of the manifestations of globalization that can be explained in this way. This concerns, among others, the biggest enlargement of the EU by ten new member states in 2004.

This opened new opportunities for the old EU companies to maximize sales. Given a significantly lower level of consumption in the new countries compared with the average level in the European Union, there were conditions that would facilitate the expansion of firms from the EU. The international demonstration effect is a factor that strongly influences the change in consumer preferences consistent with the efforts of EU companies to maximize sales.

The firms' competitiveness in the new member states was at a significantly lower level compared to firms from the old EU. This facilitated the takeover of markets from local firms. The privatization of firms has become another tool for the maximization of sales. Considering the lower costs of production and the availability of qualified employees, new member states also become new green field investment locations where supply products and final products are manufactured. The automotive industry is a good example in this context. Because of these actions, the profitability of firms and their competitiveness increase.

In 2016 the foreign trade turnover of the Visegrad Group (Poland, the Czech Republic, Slovakia, and Hungary) with Germany amounted to EUR 257 billion, which is indicative of the scale and importance of these processes (Statistisches Bundesamt (Destatis) 2017). This means that the Visegrad Group became Germany's biggest trade partner, surpassing China, which ranks second with a turnover of EUR 170 billion, and France, which ranks third with a turnover of EUR 167 billion. The transfer of profits from Poland and unpaid taxes (legal and illegal tax optimization) significantly exceeds the net transfer of funds from the EU. The enlargement of the European Union, interpreted in the light of the presented model of firm growth, lies primarily in the interest of firms of the old EU.

Production capacity constraints require significant annual investment outlays both in the home country and in the countries targeted for international expansion by large corporations (globalization processes). The above leads to the hypothesis that large corporations invest annually total depreciation. This, however, is not sufficient to achieve the NSGR. Therefore, they also make annual net investments and prefer to finance them from their retained profit, issues of corporate bonds, issues of shares, and, as the last choice, bank borrowing as the least convenient. Sustaining and disruptive innovations also require investments, which usually increase the production capacity.

The investment decisions of companies, influenced by the economic situation caused by their investments and the prevailing economic views, are affected by the belief that sustainable expansion will continue in the future. Minsky (1984) used the term a euphoric economy to refer to an economy that is characterized by such investment behaviors and predictions and pointed out its instability. It should be emphasized that the main objective of such investments is to expand the production capacity to achieve the NSGR. This is the main point of difference between my approach to a euphoric economy and Minsky's views.

I mentioned the metaphor according to which competition consists of dancing or running faster and faster, which, in the best-case scenario, may result in breathlessness. If financial markets are also characterized by the relation found between the sales growth rate and the profit growth rate, this explains the occurrence of the crisis and the behavior of such markets after the 2007–2009 global financial crisis. It was apparent how quickly the participants in the New York Stock Exchange settled into their "starting blocks" and ran faster and faster when the first signs of recovery occurred. As a result, the stock exchange indexes (e.g. Dow Jones) restored the levels from before the deep crisis unexpectedly fast. The motives and action methods of firms explain the *momentum* phenomenon, which consists of changes in asset prices in the near future according to the last tendency, even if they are overestimated.

Explaining the crisis, I can say that greed, ignorance, or even insanity and the mistakes made by individuals and institutions accused of being responsible for the crisis are not among the decisive reasons. The fact that someone is running or dancing faster and faster because others are running, or dancing faster and faster may lead not only to breathlessness but even to circulatory collapse in extreme conditions. The real sales increase rate may not be sufficient compared with the NSGR, and its actual value becomes the result of the interplay between intentions (intended activity) and possibilities (performed activity). In other words, achieving the objective of maximizing the profit (1) becomes increasingly difficult, because it requires a higher and higher NSGR unfortunately calculated from the growing sales base. I can formulate the hypothesis that, if the recovery and investment boom phase in the business cycle lasts long enough, the sales growth rates and profit growth rates should fall far below the necessary level towards the end of the phase.[16] This initiates the reversal phenomenon, whereby the previous trend in the asset price increase is reversed, triggering a crisis. This explanation of the occurrence of the upper turning point in the business cycle is different from that in the case of Minsky's moment (Minsky 1984; Chancellor 2007; Whalen 2007; Cassidy 2008; Vercelli 2009).

V. Conclusion

Lewis (2004, Kindle Location 4185) considered that: "It's hard to say new things in economics." That is why I will concentrate on presenting novel theoretical facts in this section.

The first novel theoretical fact is that the model of firm growth could be derived from the formula for the DOL. I was able to find all the other novel facts with this model. Maintaining the profit growth rate achieved in the initial period, which is the objective of a firm maximizing its profit, requires the sales growth rate to increase period by period, which is therefore called the necessary sales growth rate. Although the time paths of sales and profit are divergent, the time paths of the NSGR and ROS are convergent, and they asymptotically tend towards the initial profit growth rate and the initial contribution margin rate, respectively. These regularities are the next novel theoretical facts. These dependencies result from the cause-effect time sequence between changes in the NSGR and the DOL. The lower the DOL, the higher the NSGR necessary to maintain the previously achieved profit growth rate in subsequent periods.

The sensitivity analysis demonstrated that a decrease in the variable costs resulting from technological and organizational progress, including the application of new cost accounting methods, and a decrease in the fixed costs resulting from mergers and acquisitions do not improve the situation. On the contrary, they make the curves of the DOL fall and the time paths of the NSGR rise. The price that must be paid for increasing profit caused by a decrease in the costs is an increase in the NSGR.

Although there are many differences, the presented model of the firm resembles the Bohr model of the atom. Affected by stimuli caused by a decrease in the costs, the point of the DOL moves to the lower curve of the DOL, which belongs to a family of these curves, and the point of the NSGR moves to the time path of the NSGR, which is higher and belongs to a family of these time paths. The bigger the decrease in the costs, the stronger this stimulus and the farther the points move to more distant curves tracing the trajectory of these movements.

The model suggests explanations concerning the market and investment behaviors of firms that encounter market and production capacity constraints when achieving their objective, that

[16] Perhaps this is one of the reasons why empirical studies (Hall 1967; Mabry and Siders 1967) have not revealed a negative correlation between sales and profit.

is, maximizing their profit. This causes changes in the mentality of managers that are functional in terms of profit maximization.

I can formulate the hypothesis that large corporations not only invest annually total depreciation but also, to achieve the NSGR, make net investments and finance them primarily from retained profits, issues of corporate bonds, and issues of shares. Sustaining and disruptive innovations, which serve the same purpose, also require investments, which usually increase the production capacity.

Although there are several methods of increasing the demand, the domestic market often turns out to be insufficient to guarantee the NSGR, and international expansion becomes essential. The relations between the profit growth rate and the NSGR may be considered one of the factors explaining the processes of globalization of the economy.

The model also supplies the microeconomic foundation for macroeconomics. This leads to conclusions concerning the nature of economic recovery and the onset of a crisis. When an economy is emerging from a recession, the degree of operating leverage is very high. In such a situation, only a slight sales increase is necessary to generate a high profit growth rate. This will be a source of difficulties during the economic recovery, because in subsequent periods it will be increasingly difficult to achieve a higher and higher NSGR on a high time path that is located among a family of such paths.

This points to the hypothesis that, if the period of an economic upturn lasts long enough, companies cannot achieve an increasing NSGR, because it is too high and is calculated from the growing sales base. As a result, the profit growth rate should fall. The situation would be better if the firm curbed the increase in the sales growth rate at the beginning of the recovery. Such measures are hindered by competition. If the sales growth rate were curbed in this period, the firm's share in the market would shrink, which would be unacceptable for the managers and shareholders.

A drop in the sales growth rates and the profit growth rates are a signal, at least for some stock market investors, that the growth potential of such firms' stock prices is ending and that they should be sold. This triggers the reversal phenomenon, that is, the reversal of the previous trend of asset price increases, and this is how a crisis begins.

The main conclusion of the paper is that the market expansion of firms is not a result of the discretionary behavior of managers but rather a necessity in firms that maximize their profits. This results from the killer competition. Everyone must run or dance faster and faster, because others are running or dancing faster and faster (the orchestra is playing faster and faster).

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Appendix 1
Solution of the difference equation

It follows from the degree of operating leverage formula that the profit growth rate is equal to the product of the degree of operating leverage and the sales growth rate:

$$d_{pt+1} = d_{ot} d_{st+1} \tag{1}$$

The sales growth rate that is necessary to achieve the planned profit growth rate is equal to

$$d_{st+1} = \frac{d_{pt+1}}{d_{ot}} \tag{2}$$

where:

d_{pt+1} – the planned or achieved profit growth rate in t+1 period,

d_{ot} – the degree of operating leverage in *t* period,

d_{st+1} – the sales growth rate in t+1 period,

which is the hidden model of firm growth. To demonstrate this, the degree of operating leverage will be expressed as the relation between the contribution margin and the sales:

$$d_{ot} = \frac{d_{pt+1}}{d_{st+1}} = \frac{\frac{d_{st+1} M_t}{P_t}}{\frac{d_{st+1} S_t}{S_t}} = \frac{M_t}{P_t} \tag{3}$$

because the profit growth is

$$\Delta P_{t+1} = P_{t+1} - P_t = (1 + d_{st+1}) S_t - (1 + d_{st+1}) K_{vt} - K_{ft+1} - S_t + K_{vt} + K_{ft} =$$

$$= d_{st+1}(S_t - K_v) = d_{st+1} M_t \tag{4}$$

Formula (2) may be transformed as follows:

$$\frac{d_{pt+1}}{d_{ot}} = \frac{S_{t+1} - S_t}{S_t} \tag{5}$$

allowing the determination of the sales growth given the assumption that an enterprise wants to achieve the profit growth rate from the initial period:

$$S_{t+1} - S_t = \frac{d_p}{d_{ot}} S_t = \frac{d_p}{\frac{M_{wt}}{P_t}} S_t = \frac{d_p}{\frac{sS_t}{sS_t - K_f}} S_t = \frac{d_p}{s}(sS_t - K_f) = d_p S_t - \frac{d_p}{s} K_f \qquad (6)$$

Finally, a non-homogeneous difference equation of the first degree is derived from (6):

$$S_{t+1} - (1 + d_p) S_t = -\frac{d_p}{s} K_f \qquad (7)$$

where:

S_t – the sales in period *t* necessary to achieve in the same period initial profit rate growth d_p,

M_t – the contribution margin in period t,

$M_t = S_t - K_{vt}$

P_t – the profit in period *t*,

ΔP_{t+1} – the profit growth in period *t+1*,

K_{vt} – the total variable costs in period *t*,

K_f – the total fixed costs in period *t*.

The general solution of the first-order non-homogeneous difference equation is the sum of two components, that is, a particular integral and complementary function. The specific solution is the complementary function to the homogeneous equation:

$$S_{t+1} - (1 + d_p) S_t = 0 \qquad (8)$$

The following designation is used:

$$(1 + d_p) = a \qquad (9)$$

and

$$\mathrm{S_t} = \mathrm{Ab^t} \qquad (10)$$

(8) is substituted with (9) and (10)

$$Ab^{t+1} - aAb^t = 0 \qquad (11)$$

both sides of equation (12) are divided by $\mathrm{Ab^t}$

$$\mathrm{b - a = 0} \qquad (12)$$

Therefore

$$b = a \qquad (13)$$

Substituting (10) with (13), I arrive at the complementary function, that is, the general non-continuous solution of the equation (8):

$$S_t = Ab^t = Aa^t \tag{14}$$

which will result in a specific non-continuous solution after the *A* constant has been established. I will search for the particular integral now and attempt to find a solution with the use of the *k* constant:

if $S_t = k$ then $S_{t+1} = k$ (15)

and substitute (7) with (15)

$$k - ak = -\frac{d_p}{s} K_s \tag{16}$$

After transformation

$$k = \frac{-\frac{d_p}{s} K_f}{1-a} = -\frac{d_p K_f}{s(1-1-d_p)} = \frac{K_f}{s} = S_{BEP} \tag{17}$$

Adding complementary function (14) and particular integral (17), the general solution is:

$$S_t = A(1+d_p)^t + S_{BEP} \tag{18}$$

I eliminate the arbitrary constant A referring to the initial conditions for $t = 0$:

$$S_0 = A + S_{BEP} \tag{19}$$

Therefore, the *A* constant is

$$A = S_0 - S_{BEP} = M_s \tag{20}$$

Finally, the definite discrete solution of the first-order non-homogeneous difference equation (7) is

$$S_t = M_s (1+d_p)^t + S_{BEP} \tag{21}$$

and my objective to find a time path S_t is achieved – with (21) I can define the values of S_t in every period,

where:

s – the initial contribution margin rate,

$$s = \frac{M_0}{S_0}$$

S_{BEP} – the sales for the break-even point,

$$S_{BEP} = \frac{K_f}{s}$$

S_0 – the initial sales,

M_s – the initial safety margin.